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A note on the vorticity expulsion hypothesis

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The astrophysical importance of the hypothesis that vorticity is expelled from regions of strong turbulence is briefly reviewed. An experiment from which Gough & Lynden-Bell claimed to provide positive evidence of vorticity expulsion has been repeated. The results are reproduced here and are shown to be due to thermally driven circulation currents. No unambiguous laboratory evidence for (or against) the vorticity expulsion hypothesis therefore exists at present.

Gough & Lynden-Bell (1968) have suggested that vorticity is expelled from regions of strong turbulence just as is the case for magnetic flux (Weiss 1964). For a rotating fluid containing the axis of rotation, this would also imply expulsion of angular momentum. Such a mechanism would have profound astrophysical importance and in particular could provide a single explanation for three well established but otherwise apparently unrelated astrophysical puzzles. They are:

(*a*) Among main sequence stars, those of spectral type earlier than F 2 (i.e. with surface temperature $T > 8000\,^{\circ}\mathrm{K}$) are in general found to rotate rapidly, while those of later spectral type show little or no evidence of rotation. The change in rotational characteristics is remarkably sharp (Abt 1963), and occurs at approximately the surface temperature at which a turbulent convection zone is established in the outer layers of the star. Thus, low observed rotational velocity coincides with the existence of strong surface convection.

(*b*) The rotational velocities of red giant stars in the Hyades cluster (Kraft 1969) and moving group (Strittmatter & Norris, private communication) are much lower, if, as is almost certainly the case, these stars have evolved from typical upper main sequence stars. Once again, low surface rotation occurs after a transition in which the surface layers become convective, on this occasion in a single star.

(*c*) Dicke & Goldenberg (1967) have observed a solar oblateness (5 parts in 10^5), and attribute this to the existence of a rapidly rotating solar interior. A number of authors have pointed out that such a situation would be unstable either on a dynamical time scale (Goldreich & Schubert 1967; Fricke 1967) or on a Kelvin time scale (Howard, Moore & Spiegel 1967). However, if angular momentum is

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continuously expelled from the convective surface layers into the central regions (total angular momentum being a conserved quantity except for minor losses associated with the solar wind), a possibility exists for reconciling these two viewpoints. The actual rotational velocity of the solar surface could then be ascribed to a balance between the expulsion mechanism and the tendency for instabilities to restore angular momentum to the outer layers.

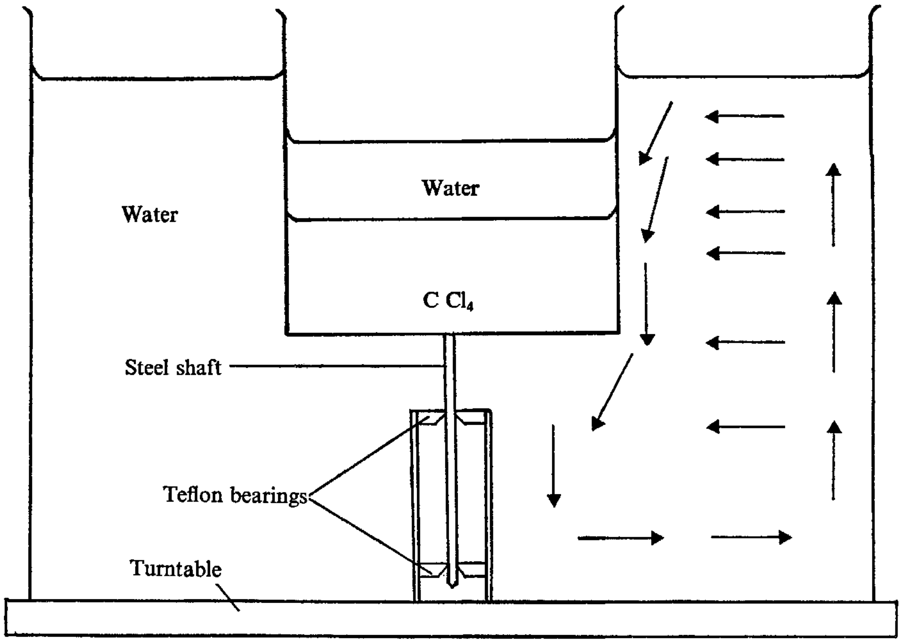

FIGURE 1. The arrangement of containers is shown schematically. Arrows indicate approximate directions of flow in the non-rotating case when the inner vessel is cooled by ice.

In view of these astrophysical applications, it is unfortunate that the vorticity expulsion hypothesis has received so little attention in the laboratory. Some early experiments by Taylor (1935) are consistent with the hypothesis, but may also be interpreted in terms of large scale circulation. More recently Bretherton & Turner (1968) have investigated the process using mechanical stirring by a grid to generate turbulence. They were unable to detect any evidence of vorticity expulsion but since the grid itself provided a powerful constraint on the motion the result is far from conclusive. On the other hand, Gough & Lynden-Bell themselves performed an experiment in which convection was created by chemical means, and claimed to have found positive evidence of vorticity expulsion. We have accordingly repeated their experiment and have succeeded in reproducing their results. These, however, seem to be predominantly due to thermal effects, and thus provide no evidence for the vorticity expulsion hypothesis. We present the following results in support of this statement.

The experimental set-up is illustrated schematically in figure 1. An inner container of turned PVC was floated in water in an outer vessel fixed to a turntable. Both vessels were cylinders of circular cross-section (diameter 12·5 cm and

28 cm respectively), their axes aligned with that of the turntable. The lateral motion of the inner container was constrained by means of a thin metal shaft attached to its base and confined to move between 'Teflon' bearings mounted centrally on the floor of the outer container. This arrangement proved remarkably friction-free, and, due to the weight of the metal shaft, was also more stable than the system used by Gough & Lynden-Bell. The rotation period of the turntable was 18 sec, and the relative position of the two containers was measured by means of a degree scale attached to the outer vessel.

The inner vessel contained 200 ml. of water and 350 ml. of carbon tetrachloride, proportions similar to those used by Gough & Lynden-Bell. Turbulent convection was created in the water by the introduction of Alkaseltzer tablets (experiment *A*). This was effected in two ways: (*a*) the tablets were mounted on a circular wire grid (various sizes were used) which was suspended above the surface of the water by means of nylon threads and rotated with the turntable; (*b*) the Alkaseltzer was crushed to granular consistency and fed through holes in a circular dish mounted over the inner container and rotating with the turntable. The method of introduction had no effect on the results, which we now summarize.

After initially decreasing during the period of strong effervescence ($\sim$ 2 min) the angular velocity of the inner container increased to a value in excess of that of the turntable and maintained this difference for a substantial period thereafter. These results are very similar to those of Gough & Lynden-Bell, but contain two disturbing features, namely, the following:

(i) The acceleration and high velocity phases occurred after strong convection had ceased.

(ii) The higher velocity was maintained in some instances for over $1\frac{1}{2}$ h. Since the experimental spin-up time for the system was approximately 10 min it is quite clear from (ii) that the inner container was being driven over a long time scale and, from (i) that vorticity expulsion could not provide the driving agency. (Slight effervescence arising from a layer of Alkaseltzer floating in the surface of the carbon tetrachloride could still be detected after 30 min. It was, however, far too weak to provide a plausible driving mechanism over the required $1\frac{1}{2}$ hr time scale). An alternative explanation of the observed behaviour was therefore sought in terms of thermally generated circulation currents.

It is well known that the Alkaseltzer reaction (between sodium bicarbonate, acetylsalicylic acid and citric acid) is endothermic; it absorbs approximately 150 calories per tablet. A similar experiment (*B*) was therefore performed, in which the Alkaseltzer tablets were replaced by ice cubes, in order to simulate the temperature effects without producing strong turbulent convection. The change in temperature in the inner container produced by 4 ice cubes ($\sim$ 20 g) and 12 Alkaseltzer tablets are compared in figure 2, and are clearly fairly similar. The corresponding changes in rotation rate are shown in figure 3, the points representing the average of two separate runs in each case. Although there is some difference in velocity during the period of strong convection (see discussion below), the subsequent behaviour is remarkably similar in both cases. We therefore conclude that the increase in velocity is due to thermal effects and provides no positive evidence for vorticity expulsion.

To isolate the cause of the velocity increase the following tests were carried out.

(*a*) With the turntable stationary, ice was added to the fluid in the inner container. Drops of a neutrally buoyant dye were then introduced at various locations in the outer fluids, in order to trace the circulation streamlines. The general directions of flow are shown in figure 1. As expected, the water moved slowly inwards in the upper levels, and turned downwards near the cool wall of the inner container. The return flow was very much more rapid occurring almost entirely in a thin layer ($\sim$ 1 cm deep) located about 2 cm above the floor of the outer vessel. This circulation is precisely of the type which would be driven by cooling at the inner container wall, and which would result in an increase of angular velocity towards the centre when the system rotates.

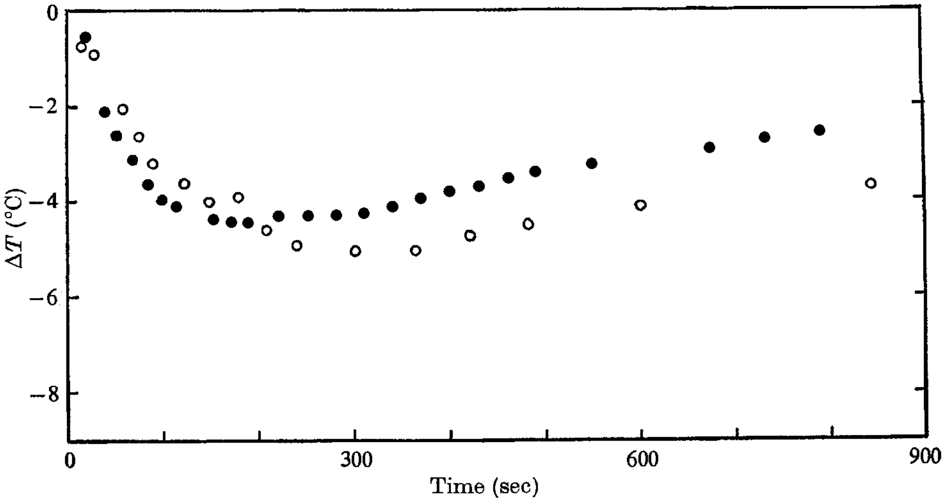

FIGURE 2. Temperature changes in the inner container caused by the introduction of 12 Alkaseltzer tablets and 20 g of ice are compared: ●, Alkaseltzer tablets; ○, ice.

(*b*) Experiment *A* was repeated with the modification that the diameter of the outer container was reduced to 16 cm (experiment *C*). The results are shown in figure 3. Although comparable with experiment *A* in the strongly effervescent stage, the relative velocity was much reduced in the subsequent phases. Such a result is clearly consistent with a reduction in the horizontal scale of circulation in the outer container.

(*c*) Experiment *A* was repeated with the rotation period of the turntable reduced to 8 sec (experiment *D*) and the results are again shown in figure 3. Once more, initial motion is similar to that in experiment *A*, but the subsequent relative velocity reduced. This is consistent with a reduction of the horizontal length scale of circulation at the higher rotational velocity as a result of stronger Coriolis forces (the Taylor–Proudman theorem).

We conclude, therefore, that thermally driven circulation currents in the outer container are responsible for the increase in rotational velocity of the inner vessel once convection has ceased. We would, however, draw attention to the motion in the initial phases. A strong reverse motion occurred whenever

Alkaseltzer was used to create convection, but was barely detectable when ice was used. A marked difference in motion in the fluid in the inner container was also apparent. During effervescence a strong vortex was established subsiding again after violent convection ceased. The consequent lower velocity at the wall

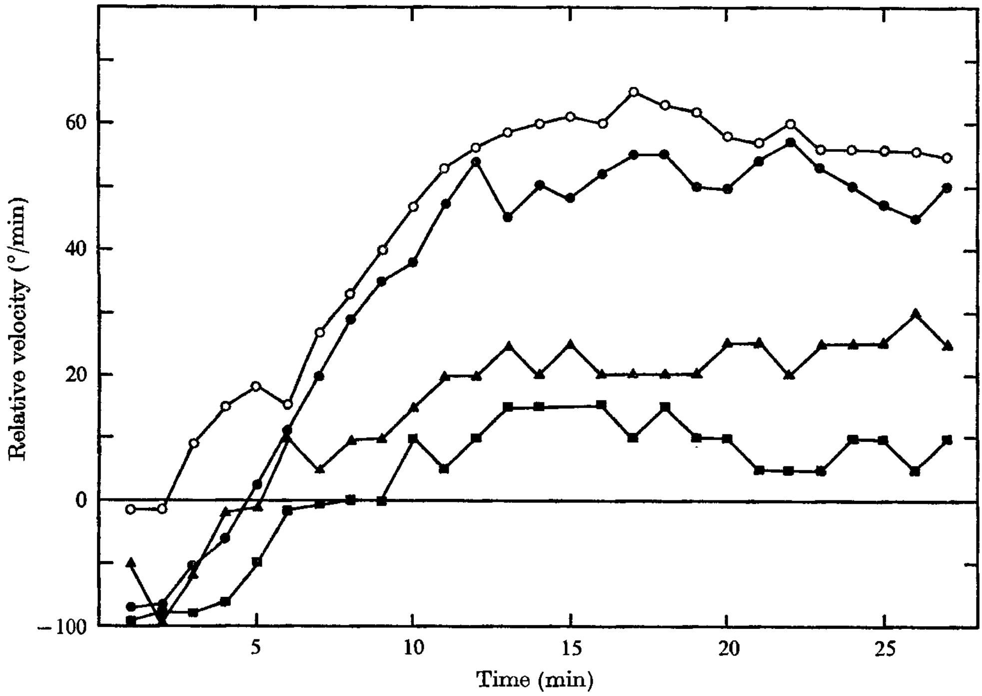

FIGURE 3. The excess angular velocity of the inner container is shown as a function of time for experiments *A*, *B*, *C*, and *D* (note change in scale for negative relative velocities): ●, *A*; ○, *B*; ▲, *C*; ■, *D*.

could clearly account for the initial sharp acceleration of the inner container in the Alkaseltzer experiments. On the other hand, neither a vortex nor a marked deceleration were detected in the ice experiments. (We should note that Gough & Lynden-Bell found only slight initial deceleration of the inner beaker in their experiment. This was almost certainly due to the presence in the convecting fluid of a wire frame supported by a nail attached to the floor of the beaker. Although this grid was supposed to rotate freely, friction was so great as to effectively force it to rotate with the beaker (Gough, private communication). Hence, the grid would not only tend to prevent the formation of a central vortex, but would also transmit any torques from it to the beaker, thus preventing any marked deceleration.) The initial retrograde motion of the inner container did not show any obvious correlation with turntable rotation speed.

The origin of the vortex may, in our view, be either (i) vigorous meridional circulation arising from the effervescence, or (ii) explusion of vorticity by convection resulting, at least initially in an attempt to establish a uniform distribution of angular momentum per unit mass. The former explanation receives strong support from the carbonated water tornado experiments of Turner & Lilly

(1963), although we note that their results were obtained in relatively deep fluid layers, and for the case in which the rotation period (0·66 sec) was short compared to the convective eddy time. Both conditions are reversed in the present study, and for this reason attempts to use dye to detect meridional circulation in the inner container during the initial phases proved fruitless. Both conditions (shallow layer and strong convection) would tend to favour the vorticity expulsion mechanism. Also, in favour of (ii), we note that the strength of the vortex was undiminished (as judged by retrograde motion of the inner container) when Alkaseltzer was introduced in granular form and distributed evenly over the fluid surface, thus reducing the possibility of circulation driven by localized sources of convection.

Our results may therefore still contain a suggestion that vorticity expulsion by convection can occur. The entire experimental technique is unfortunately too crude to obtain definitive results, and alternative approaches are therefore being investigated. At present, however, it appears that the astrophysical evidence in favour of the vorticity expulsion hypothesis is very much stronger than that so far obtained in the laboratory. It is our hope that this communication will stimulate laboratory investigations that will settle the issue.

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